Issues of the Transition to University Mathematics

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PowerPoint available at www.macalester.edu/~bressoud/talks
Remarks by President Obama at the National Academy of Sciences, April, 2009:

“… our work does not end with a high school diploma. For decades, we led the world in educational attainment, and as a consequence we led the world in economic growth … But in this new economy, we've come to trail other nations in graduation rates, in educational achievement, and in the production of scientists and engineers.”
1. Evidence of a problem in the preparation of mathematicians, scientists, and engineers.

2. The effect of AP Calculus on this problem.

3. Components of the solution:

   a) Gather better information about the problem.

   b) Work on PreK-12 mathematics education.

   c) Work on first-year college mathematics.
Bachelors Degrees, math-intensive majors

NCES data
Math-intensive majors as % of Bachelors Degrees

- Engineering
- Physical Science
- Math & Stat

NCES data
Bachelors Degrees, math-intensive majors, as % of 20–24 year-olds

NCES & US Census data
Fall enrollments (thousands), Calculus and above, 4-year undergraduate programs

CBMS data
Fall enrollments, Calculus and above, as percentage of 20–24 year-olds
Increasingly, the students who do not have to take mathematics, don’t.
Hispanic students as % of Bachelors Degrees in selected fields

- Engineering
- Mathematics
- Physical Sciences

NCES data
There were 1,089 Bachelors in Math or Stat earned by African-Americans in 1997. By 2008, that number was down to 818.
186,000 students entered four-year undergraduate programs with the intention of majoring in engineering, a physical science, mathematics, or statistics.

Approximately 600,000 of the entering students passed a class in calculus while in high school.
Over 600,000 students studied calculus in high school this year, roughly 1/3 of the 1.8 million who will go directly from HS to college.

Slightly over 150,000 of these students earned and used credit for Calculus I.
Fall Enrollments, Calculus II (thousands)

- 4-year colleges
- 2-year colleges

CBMS data
All evidence suggests that calculus in high school works well for most of the roughly 25% who earn and use their college credit.

What about the other 75%?
Of the high school students who graduated in 1992 and studied “calculus” while in high school, 31% took *precalculus* in college, and a further 32% took *no calculus* in college.

Of the high school students who graduated in 2004 and studied “calculus” while in high school, 17% took *remedial mathematics* in college.

NCES, NELS:88 and ELS:2002/06 data.
Of the high school students who graduated in 1992 and studied “calculus” while in high school, 31% took precalculus in college, and a further 32% took no calculus in college.

We must have clear, enforced guidelines for what it means to be ready for calculus in high school.

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NCES, NELS:88 and ELS:2002/06 data.
What happens to those who take calculus in High School?

What are the strengths and weaknesses of incoming students?

How well are existing placement programs and courses serving our students?
Of students who took pre-calculus and
• Their declared major required at least one semester of calculus, and
• They earned an A in pre-calculus,

43% chose not to enroll in calculus.

Pat Thompson, lead author
During the period fall 2001 through fall 2006, 43% of engineering majors, 54% of mathematics majors, 51% of physical science majors, and 50% of technology majors who enrolled in Calculus I at ASU and whose intended majors required Calculus II never earned credit for Calculus II.
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The point is that ASU gathered this information, and they are now doing something about it.
Phil Sadler

*Factors Influencing College Success in Mathematics*

Controlling for socio-economic factors, what aspects of high school mathematics prepare students for success in Calculus I?
Bressoud, Carlson, Pearson, Rasmussen: *Characteristics of Successful Programs in College Calculus*

College factors that influence success in Calculus I and case study analysis of successful programs.
SOLUTIONS: Improve PreK-12 Math

Bill McCallum

Intel Math: Mathematics Professional Development for K-8 Teachers
SOLUTIONS: Improve PreK-12 Math

NebraskaMATH
A Partnership to Improve Mathematics Achievement

Jim Lewis

NJ Partnership for Excellence in Middle School Mathematics

Amy Cohen
SOLUTIONS: Improve PreK-12 Math

SIGMAA on Circles

The participants of the first Math Teachers’ Circle at AIM, Summer 2006.
SOLUTIONS: Improve first-year college mathematics

Place students in appropriate courses.

College Placement Testing in Mathematics

Educational accomplishments in mathematics often exert a strong influence on career accomplishments. College-level mathematics study must build on and extend prior experiences. Students entering higher education have diverse preparations for college mathematics due to many factors including academic background, time since high school graduation, age, and work experience. As a result, mathematics departments in colleges and universities have difficulty in placing students in their first college mathematics courses by using only data such as high school rank-in-class, grade point average, or record of high school mathematics courses.

Placement tests can be an effective component of a comprehensive placement process. However, it is important to recognize that the development of testing instruments is a nontrivial process. The Mathematical Association of America recommends that college placement tests in mathematics should:

MEASURE DEVELOPED MATHEMATICAL REASONING SKILLS. College admission tests such as the SAT or ACT measure students’ general readiness for college, whereas placement tests seek to measure students’ knowledge and skills that are prerequisite for specific entry-level college mathematics courses. Nationally administered tests such as SAT and ACT measure a broad range of quantitative skills, and this measure is often too general to distinguish between readiness for entry-level mathematics courses such as college algebra, trigonometry, pre-calculus, and calculus. Therefore, very often, high school record and admission test scores need to be supplemented to make decisions about placing entering students into their initial mathematics courses.

EMPHASIZE REALISTIC AND CURRENT EXPECTATIONS. Placement tests should not reflect unsolvable expectations in mathematics preparation in the secondary schools. Placement tests must be carefully reviewed as more is learned about what contributes to success in post secondary education and in light of changes in content and effectiveness of pre-collegiate mathematics programs.

AVOID SINGULAR FOCUS ON COMPUTATIONAL SKILLS. Good placement tests assess computational skills in unexpected contexts and a balance of procedural fluency, conceptual understanding, and strategic reasoning.

INCORPORATE APPROPRIATE TECHNOLOGY. Calculators and computers are an integral part of most pre-collegiate mathematics instruction. Even though prerequisite skills for a college mathematics course can be assessed without computers or calculators, students may be more comfortable working on a placement test in the familiar environment that includes use of technology. Therefore, calculators and computers should be considered for use in placement testing programs.

USE APPROPRIATE TESTING METHODS. Great care should be used in the design and administration of placement test programs. Informed consultants and helpful literature should be utilized in the design of placement test programs.

Further information on design of effective college placement programs for mathematics can be obtained from the Mathematical Association of America, 1539 Eighteenth Street, NW, Washington, DC 20036.

— MAA Board of Governors
August 2010
SOLUTIONS: Improve first-year college mathematics

Use online resources to address individual student weaknesses.
SOLUTIONS: Improve first-year college mathematics

MA 103: Mathematical Modeling and Introduction to Calculus.
The course lays the foundation for calculus and differential equations through difference equations

This course has now been in place for twenty years. A similar course at Macalester is over 5 years old.
MTBI supports the development of students through educational, research and mentorship activities from the undergraduate to the postdoctoral level.

Carlos Castillo-Chavez
“The mathematics profession as a whole has seriously underestimated the difficulty of teaching mathematics.”

Ramesh Gangolli
MER Workshop
May 31, 1991

With thanks to Susanna Epp for preserving this quote.

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