

COURSE SCALE LEARNING GOALS

Upper-Division Electricity & Magnetism, Physics 3310, University of Colorado – Boulder
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These learning goals were compiled with intensive faculty input.

This list represents what we want students to be able to *do* at the end of the course (as opposed to what *content* is expected to be covered, as in a syllabus).

All course materials are at http://www.colorado.edu/sei/departments/physics_3310.htm

1. **Math/physics connection:** Students should be able to translate a physical description of a junior-level electromagnetism problem to a mathematical equation necessary to solve it. Students should be able to explain the physical meaning of the formal and/or mathematical formulation of and/or solution to a junior-level electromagnetism problem. Students should be able to achieve physical insight through the mathematics of a problem.
2. **Visualize the problem:** Students should be able to sketch the physical parameters of a problem (e.g., E or B field, distribution of charges, polarization), as appropriate for a particular problem.
3. **Organized knowledge:** Students should be able to articulate the big ideas from each chapter, section, and/or lecture, thus indicating that they have organized their content knowledge. They should be able to filter this knowledge to access the information that they need to apply to a particular physical problem, and make connections/links between different concepts.
4. **Communication.** Students should be able to justify and explain their thinking and/or approach to a problem or physical situation, in either written or oral form.
5. **Problem-solving techniques:** Students should be able to choose and apply the problem-solving technique that is appropriate to a particular problem. This indicates that they have learned the essential features of different problem-solving techniques (eg., separation of variables, method of images, direct integration). They should be able to apply these problem-solving approaches to novel contexts (i.e., to solve problems which do not map directly to those in the book), indicating that they understand the essential features of the technique rather than just the mechanics of its application. They should be able to justify their approach for solving a particular problem.
 - ...5a. **Approximations:** Students should be able to recognize when approximations are useful, and use them effectively (eg., when the observer is very far away from or very close to the source). Students should be able to indicate how many terms of a series solution must be retained to obtain a solution of a given order.
 - ...5b. **Series expansions:** Students should be able to recognize when a series expansion is appropriate to approximate a solution, and complete a Taylor Series to two terms.

...5c. Symmetries: Students should be able to recognize symmetries and be able to take advantage of them in order to choose the appropriate method for solving a problem (eg., when to use Gauss' Law, when to use separation of variables in a particular coordinate system).

...5d. Integration: Given a physical situation, students should be able to write down the required partial differential equation, or line, surface or volume integral, and correctly calculate the answer.

...5e. Superposition: Students should recognize that – in a linear system – the solutions may be formed by superposition of components.

6. **Problem-solving strategy:** Students should be able to draw upon an organized set of content knowledge (LG#3), and apply problem-solving techniques (LG#4) to that knowledge in order to organize and carry out long analyses of physical problems. They should be able to connect the pieces of a problem to reach the final solution. They should recognize that wrong turns are valuable in learning the material, be able to recover from their mistakes, and persist in working to the solution even though they don't necessarily see the path to the solution when they begin the problem. Students should be able to articulate what it is that needs to be solved in a particular problem and know when they have solved it.
7. **Expecting and checking solution:** When appropriate for a given problem, students should be able to articulate their expectations for the solution to a problem, such as direction of the field, dependence on coordinate variables, and behavior at large distances. For all problems, students should be able to justify the reasonableness of a solution they have reached, by methods such as checking the symmetry of the solution, looking at limiting or special cases, relating to cases with known solutions, checking units, dimensional analysis, and/or checking the scale/order of magnitude of the answer.
8. **Intellectual maturity:** Students should accept responsibility for their own learning. They should be aware of what they do and don't understand about physical phenomena and classes of problem. This is evidenced by asking sophisticated, specific questions; being able to articulate where in a problem they experienced difficulty; and take action to move beyond that difficulty.
9. **Maxwell's Equations.** Students should see the various laws in the course as part of the coherent field theory of electromagnetism; ie., Maxwell's equations.
10. **Build on Earlier Material.** Students should deepen their understanding of Phys 1120 material. I.e., the course should build on earlier material.

OVERALL COURSE OBJECTIVES: CALCULATION AND COMPUTATION

Students will be able to:

- Compute gradient, divergence, curl, and Laplacian
- Evaluate line, surface, and volume integrals
- Apply the fundamental theorem for divergences (Gauss' Theorem) in specific situations
- Apply the fundamental theorem for curls (Stoke's Theorem) in specific situations
- Apply Coulomb's Law and superposition principle to calculate electric field due to a continuous charge distribution (uniformly charged line segment, circular or square loop, sphere, etc.)
- Apply Gauss' Law to compute electric field due to symmetric charge distribution
- Calculate electric field from electric potential and vice versa
- Compute the potential of a localized charge distribution
- Determine the surface charge distribution on a conductor in equilibrium
- Use method of images to determine the potential in a region
- Solve Laplace's equation to determine the potential in a region given the potential or charge distribution at the boundary (Cartesian, spherical and cylindrical coordinates)
- Use multipole expansion to determine the leading contribution to the potential at large distances from a charge distribution
- Calculate the field of a polarized object
- Find the location and amount of all bound charges in a dielectric material
- Apply Biot-Savart Law and Ampere's Law to compute magnetic field due to a current distribution
- Compute vector potential of a localized current distribution using multipole expansion
- Calculate magnetic field from the vector potential
- Calculate the field of a magnetized object
- Compute the bound surface and volume currents in a magnetized object
- Compute magnetization, H field, susceptibility and permeability

CHAPTER SCALE LEARNING GOALS

CHAPTER 1: Vector analysis

TOPICS

- Div, grad, curl
- Line, surface, volume integrals
- Curvilinear coordinates
- Dirac delta function
- Vector fields (potentials)

PREREQUISITES: Students should already be able to...

1. Be able to compute correctly div, grad and curl in rectangular coordinates for any test function
2. Do a path integral along a specific path -- eg. Griffiths 2.20
3. Be able to expand $1/1+e$ and $1/1-e$ when e is very small (Taylor series).

Students should be able to:

1. Evaluate the integral from negative infinity to infinity of the delta function, $d(x)$
2. Evaluate the 3-dimensional divergence of $1/r^2$ in the r -hat direction [$4\pi \delta^3(\mathbf{r})$]
3. Evaluate the integral of a function times the delta function
4. Be able to evaluate the integral of $1/(x-r)^{(3/2)}dx$
5. Give a geometrical description of the divergence theorem, and fundamental theorem for curls.
6. Change a multidimensional integral in Cartesian coordinates to one in another coordinate system using the Jacobian.

CHAPTER 2: Electrostatics

TOPICS

- Electric field, Coulomb's law
- Gauss' Law, divergence and curl of E
- Potential
- Poisson & Laplace equation
- Work & energy
- Conductors

PREREQUISITES: Students should already be able to...

1. State Gauss' Law and construct the 3 Gaussian surfaces (sphere, cylinder, pillbox).
2. Use Cartesian, spherical and cylindrical coordinates appropriately when constructing integrals and surface and volume elements.

Electric Field

- Students should be able to state Coulomb's Law and use it to solve for E above a line of charge, a loop of charge, and a circular disk of charge.
- Students should be able to solve surface and line integrals in curvilinear coordinates (when given the appropriate formulas, as in the inner-front cover of Griffiths).

Divergence and Curl of E ; Gauss' Law

1. Students should recognize when Gauss' Law is the appropriate way to solve a problem (by recognizing cases of symmetry; and by recognizing limiting cases, such as being very close to a charged body).

2. Students should be able to recognize that E comes out of the Gaussian integral only if it is constant along the Gaussian surface.
3. Students should be able to recognize Gauss' Law in differential form and use it to solve for the charge density ρ given an electric field E .

Electric Potential

1. Students should be able to state two ways of calculating the potential (via the charge distribution and via the E -field); indicate which is the appropriate formulation in different situations; and correctly evaluate it via the chosen formulation.
2. Students should be able to calculate the electric field of a charge configuration or region of space when given its potential.
3. Students should be able to state that potential is force per unit charge, and give a conceptual description of V and its relationship to energy.
4. Students should be able to explain why we can define a vector potential V (because the curl of E is zero and E is a conservative field).
5. Students should be able to defend the choice of a suitable reference point for evaluating V (generally infinity or zero), and explain why we have the freedom to choose this reference point (because V is arbitrary with respect to a scalar – its slope is important, not its absolute value)

Work & Energy

- Students should be able to calculate the energy stored in a continuous charge distribution when given the appropriate formula
- Students should be able to explain in words what this energy represents.

Conductors

- Students should be able to sketch the induced charge distribution on a conductor placed in an electric field.
- Students should be able to explain what happens to a conductor when it is placed in an electric field, and sketch the E field inside and outside a conducting sphere placed in an electric field.
- Students should be able to explain how conductors shield electric fields, and describe the consequences of this fact in particular physical problems (e.g., conductors with cavities).
- Students should be able to state that conductors are equipotentials, that $E=0$ inside a conductor, that E is perpendicular to the surface of a conductor (just outside the conductor), and that all charge resides on the surface of a conductor.

CHAPTER 3: Special Techniques

TOPICS:

1. Laplace's equation
2. Boundary conditions and uniqueness
3. Method of images
4. Separation of variables in Cartesian and spherical
5. Multipole expansion

PREREQUISITES: Students should already be able to...

- Recognize the wave equation in Cartesian coordinates, and state that e^{ikx} is a solution
- Recognize the solution to separation of variables in Cartesian coordinates.

- Recognize that a function can be expanded in terms of a complete basis, such as sin and cos.
- State that conductors are equipotentials.

Laplace's equation

- Students should recognize that the solution to Laplace's equation is unique.

Method of Images.

- Students should realize when the method of images is applicable and be able to solve simple cases.
- Students should be able to explain the difference between the physical situation (surface charges) and the mathematical setup (image charges).

Separation of variables/boundary value problems

- Students should be able to state the appropriate boundary conditions on V in electrostatics
- Students should recognize where separation of variables is applicable and what coordinate system is appropriate to separate in.
- Students should be able to outline the general steps necessary for solving a problem using separation of variables.
- Students should be able to state what the basis sets are for separation of variables in Cartesian and spherical coordinates (ie., exponentials, sin/cos, and Legendre polynomials.)
- Students should be able to apply the physics and symmetry of a problem to state appropriate boundary conditions.
- Students should be able to solve for the coefficients in the series solution for V , by expanding the potential or charge distribution in terms of special functions and using the completeness/orthogonality of the special functions, and express the final answer as a sum over these coefficients.

Multipole expansions

- Students should be able to explain when and why approximate potentials are useful.
- Students should be able to identify and calculate the lowest-order term in the monopole expansion (i.e., the first non-zero term).
- Students should be able to sketch the direction and calculate the dipole moment of a given charge distribution.

CHAPTER 4: Electric Fields in Matter

TOPICS

- Polarization & dielectrics
- Field of polarized object (bound charges, field inside dielectric)
- Electric displacement
- Linear dielectrics: Susceptibility, permittivity, dielectric constant
- Boundary value problems with dielectrics

Polarization and dielectrics

1. Students should be able to go between two representations of dipoles – as point charges, and as generalized dipole vectors – for simple charge configurations.
2. Students should be able to calculate the dipole moment of a simple charge distribution.
3. Students should be able to name 4 similarities and differences between a conductor and a dielectric (both shield E , conductor shields E completely, dielectric shields via fixed dipoles, conductor shields via mobile electrons).
4. Students should be able to predict whether a particular pattern of polarization will result in bound surface and/or volume charge

5. Students should be able to explain the physical origin of bound charge.

Field of a polarized object

1. Students should be able to sketch the E field inside and outside a dielectric sphere placed in an electric field.
2. Students should be able to explain what happens to a dielectric, when it is placed in an electric field.
3. Students should be able to explain the difference between free and bound charge.
4. Students should be able to identify the appropriate boundary conditions on D given its relationship to E and Qf.

Electric displacement

- Students should be able to sketch the direction of D, P, and E for simple problems involving dielectrics
- Students should be able to calculate the E field inside a dielectric when given epsilon and the free charge on the dielectric.

Linear dielectrics

- Students should be able to articulate the difference between a linear and nonlinear dielectric.
- Students should be able to write down Maxwell's equations (for electrostatics) in matter, when given the appropriate equations in vacuum.
- Students should be able to state the differences in boundary conditions for fields (D) in matter versus a conductor (D).

CHAPTER 5: Magnetostatics

TOPICS

- Currents and charge density
- Magnetic fields and forces (Lorentz force law)
- Biot-Savart law
- Divergence and curl of B (Ampere's Law)
- Magnetic vector potential

PREREQUISITES: Students should already be able to...

1. Write down Lorentz force law
2. Know the right-hand rule and how to apply it

Currents and charge density

1. Students should be able to calculate current density J given the current I, and know the units for each.
2. Students should be able to explain, in words, what the charge continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0$ means.
3. Students should be able to state the vector form of Ohm's Law ($J = \sigma E$) and when it applies.
4. Students should be able to calculate the current I, K and J in terms of the velocity of the particle or in terms of each other.

Magnetic fields and forces

1. Students should be able to describe the trajectory of a charged particle in a given magnetic field.
2. Students should be able to sketch the B field around a current distribution, and explain why any components of the field are zero.

3. Students should be able to explain why the magnetic field does no work.

Biot-Savart Law

- Students should be able to state when the Biot-Savart Law applies (magnetostatics; steady currents, $dp/dt=0$).
- Students should be able to compare similarities and differences between the Biot-Savart law and Coulomb's Law.
- Students should be able to choose when to use Biot-Savart Law versus Ampere's Law to calculate B fields, and to complete the calculation in simple cases.

Divergence and curl of B (Ampere's Law)

1. Students should be able to draw appropriate Amperian loops for the cases in which symmetry allows for solution of the B field using Ampere's Law (ie., infinite wire, infinite plane, infinite solenoid, toroids), and calculate I_{enc} .
2. Students should be able to make comparisons between E and B in Maxwell's equations (what exactly do we want?)

Magnetic vector potential

1. Students should be able to explain why the potential A is a vector for magnetostatics, whereas it's a scalar (V) in electrostatics. Ie., that the source of magnetic fields is a vector, not a scalar.
2. Students should recognize that A does not have a physical interpretation similar to V.

CHAPTER 6: Magnetic Fields in Matter

TOPICS

1. Magnetization – diamagnets, paramagnets, ferromagnets
2. Field of magnetized object (bound currents)
3. Auxiliary field H
4. Linear and nonlinear media: susceptibility, permeability

Magnetization

1. Students should be able to calculate the torque on a magnetic dipole in a magnetic field.
2. Students should be able to explain the difference between para, dia, and ferromagnets, and predict how they will behave in a magnetic field.

The Field of a magnetized object

1. Students should be able to predict whether a particular magnetization will result in a bound surface and/or volume current, for simple magnetizations.
2. Students should be able to give a physical interpretation of bound surface and volume current, using Stokes' Theorem.

Auxiliary field H

1. Students should be able to calculate H when given B or M
2. Students should be able to use H to calculate B when given J_f for an appropriately symmetric current distribution.
3. Students should be able to articulate in which physical situations it is useful to use H.
4. Students should be able to identify the appropriate boundary conditions on H given its relationship to M and K_f .