



Background

A worked example is an example that involves a step-by-step solution to a problem. It can be presented in textual, graphical, video or face-to-face format (latter is sometimes called "expert modelling").

A self-explanation or a self-generated explanation is an explanation of presented instruction that integrates the presented information with background knowledge and fills in tacit inferences.

A prompted self-explanation is a process by which, when given a piece of material to study, students are given external cues eliciting self-explaining.

Many studies show the effectiveness of both worked examples (Renal et al.), selfexplanations (Chi at al.). However, there is a large amount of variance in the amount and quality of individually produced self-explanations. Prompting selfexplanations is a more reliable technique and has been shown to work with:

- verbal prompts (Chi et al., 1994),
- computer-generated prompts (McNamara, 2004; Aleven and Koedinger, 2002; Hausmann and Chi, 2002),
- prompts embedded in the learning materials (Hausmann and VanLehn, 2007).

So far, none of the studies were applied to a service first-year calculus course.

Course

MATH 110 is a two-term, six-credit course in differential calculus that covers the same calculus content as the one-term courses, but with additional material designed to strengthen understanding of essential pre-calculus topics.

This course is meant for students who do not satisfy the prerequisites for one-term calculus courses and normally students with Pre-Calculus 12 grades higher than 85% are not permitted to take MATH 110.

Implementation

Why? Pilot study to test out materials and figure out controls for next year's implementation.

Who? One section of MATH 110 with \sim 50 students.

What? Introduce prompted self-explanations as an active learning technique.

How? Provide worked examples and ask students to explain particular steps.

How often? One hour a week in class (worksheets) plus one question on every assignment.

Using prompted self-explanations in first-year calculus Kseniya Garaschuk and Costanza Piccolo {kseniya, costanza}@math.ubc.ca

Sample materials

Problem 1: A spotlight on the ground shines on a wall 12 meters away. If a two meter man walks from the spotlight to the wall at a speed of 1.6 meters per second, how fast is the length of his shadow on the wall decreasing when he is 4 meters from the wall?		
Solution.	· · · · · · · · · · · · · · · · · · ·	
Let us first draw a diagram and label it.	Hand ir argumer drawing at the to	
light 2	1. Th by Ye	
x wall 12	2. Be (i) yo	
Let x be the distance from the man to the light and let y be the height of his shadow. We are given that $\frac{dx}{dt} = 1.6$; we are asked to find $\frac{dy}{dt}$ when $x = 8$.	(ii (ii Pr	
$\frac{x}{2} = \frac{12}{y} \longrightarrow $ Why?	wi sic Sc	
$xy = 24$ $\frac{d}{(mi)} = \frac{d}{(24)}$	Im How did the we get The this? The	
$\frac{d}{dt}(xy) = \frac{d}{dt}(24)$		
$\frac{dx}{dt}y + x\frac{dy}{dt} = 0 \qquad \rightarrow \qquad \text{Why? What rule(s) did you use?}$	5. Why are The second we doing The	
$\frac{dy}{dt} = -\frac{y}{dt}\frac{dx}{dt}$	3. Fin loc	
$dt \qquad x \ dt$ When $x = 8$, we have $y = 3$. \rightarrow Why?	4. A of	
	5. Sheeq	
Therefore, $\frac{dy}{dt} = -\frac{3}{8}(1.6) = -0.6$ meters per second. Does the answer make sense? Explain.	6. A a r wa on	

Sample student work

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Liuth 110: Hand-in Assignm	vent 17	
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estimate of a value 1	sino derivative	
Calculus. The estimate	s based on	
Known inline for imma	montal injunes and	
the slope the function	takas an after hetar	
this known inter	lates on afreque,	(a) Use a
to estimate the value of ((a))		(b) Draw
Change A set of flory		may
1 actionate Miles 2	find the difference between a in x	(c) Deter
approximate $f(x) = 0.9$	to be able to find the change in	
(1) - 2 - Value to	1 2 + 1 - 0 - 0	a) f(
+1(1)-1 = 1 base The	1 = 0.4-1=-0.1-0x	
Q	fot. f'(x) dx de	f(a)
f'(x) = 2x - f'(x) - 2		fía
f(x) = bx - f(1) = b	2×-0.1 =-9.2	, (*.
find the derivative plug-in	known $1 + (-0.2) = 0.8 = f_2$	
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	C int linear	
	$t_1 + \alpha_1$ approximation.	
list of information was mod.	add the known value	
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n full solutions to the questions below. Make sure you justify all your work and include complete ents and explanations. Your asswers must be clear and neatly written, as well as legible (no tiny s or micro-handwriting please!). Your answers must be stapled, with your name and student number top of each page. 'his week we learned how to solve optimization problems. Explain in your own words what we mean optimization problems", then provide an example of an optimization problem with a full solution.

our example must be an original problem (not from webwork, calculus textbooks, Google, etc.) elow is an optimization problem with solution. Your task is to draw a sketch representing the situation described in the problem (you should label quantities on our sketch consistently with the solution provided)

) answer the handwritten questions. i) show a different way to derive the equation for the area of the rectangle

roblem Find the dimensions of the rectangle of maximum area that can be inscribed in a right triangle th sides of length 3 and 4 and hypothenuse of length 5, if two sides of the rectangle lie along the two les of the triangle. Make sure you justify that your answer is a maximum.

agine the right triangle drawn with the x-axis and y-axis forming the two perpendicular sides and $\lim_{x \to -\frac{4}{3}x + 4}$ forms the hypotenus The area of the rectangle of dimensions a, b inscribed in the triangle is 2. How did we get this? $f(a) = (a(-\frac{4}{3}a+4)) = (-\frac{4}{3}a^2 + 4a)^3$. How does this step help us?

6. Why ? Then $f'(a) = -\frac{8}{3}a + 4$. (The critical number of f is $a = \frac{3}{2}$.) A, why ie check that f'(a) > 0 for $0 < a < \frac{3}{2}$ and f'(a) < 0 for $\frac{3}{2} < a < 3$. Thus f(a) is maximized at $a = \frac{3}{2}$ hus the dimensions of the largest rectangle inscribed in such triangle are $a = \frac{3}{2}$ and b = 2. =, those did we are this. nd the length of the shortest ladder than can extend from a vertical wall, over a fence 2m high cated 1m away from the wall, to a point on the ground beyond the fence. cylindrical can is to be made to hold 1 litre of oil. Find the dimensions that will minimize the cost the metal to manufacture the can.

ow that of all the isosceles triangles with a given perimeter, the one with the greatest area is boat on the ocean is 4 km from the nearest point on a straight shoreline; that point is 6 km from restaurant on the shore. A woman plans to row the boat straight to a point on the shore and the

lk along the shore to the restaurant. If she walks at 3 km/hr and rows at 2 km/hr, at which poin n the shore should she land to minimize the total travel time? If she walks at 3 km/hr, what is the minimum speed at which she must row so that the quickest way to the restaurant is to row directly (with no walking)

Math 110 Quiz 19 - April 8, 2015
now all your work. No calculators, nor books/notes are allowed.
iate linear approximation to approximate $\sqrt[3]{8.1}$.
to explain how the approximation above is obtained. Your graph should be clear and acale.
ther your approximation is an underestimate or an overestimate and explain why.
$T(a)(x-a)$ where $k = 8.1$, $a = 8$, and $f(x) = \sqrt[3]{x}$
$ \begin{cases} 2 \\ (x)^{-2/3} \end{cases} $ plug into $f(a) + f'(a)(x-a) + o find equation $
$\frac{1}{3^{2}} (4) = 2 + (4)(8.1 - 8)$
imation = 2 + 44(4)(.1)
$\frac{1}{2} \left[\frac{1}{2} + 1$
tangent - 2.1
To determine concauty f'a)=1/3 X ^{-2/3}
$f'(a) = \frac{1}{2} \cdot \frac{2}{2} \cdot \frac{1}{2}$
881 = negative
pproximation is on the angent line to 8 k=8.
erestimation, as one can tell by Looking at the sits concavity. The pointer = 8.1 is higher on the
line at point a= 8 than the function. ong mac.
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Encouraging preliminary results

We used Mathematics Attitudes and Perceptions Survey (MAPS) to study the results. The basis for comparison was MAPS data collected in MATH 110 in April 2012. Assuming the population of Math 110 students does not significantly change through the years, we see some non-trivial improvements. Below we present the data for questions with the most significant improvement:



Q2: After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic. Q10: Understanding math means being able to recall something you've read or been shown. Q15: In math, it is important for me to make sense out of formulas and procedures before I use them. Q19: I often have difficulty organizing my thoughts during a math test. Q26: School mathematics has little to do with what I experience in the real world. Q30: If I get stuck on a math problem, there is no chance that I will figure it out on my own.

Overall MAPS average agreement with the experts: 27% for the 2015 group versus 11% for the 2012 group.

Student comments

"Explaining worked solutions was very useful in my understanding how to solve the problem as well as learning where I was going wrong. Having to explain solutions made sure that I had to be confident I knew why I did each step which was helpful."

"I was doubtful on the usefulness at the start but going through the problems stepby-step with teacher and students helps."

"Showing how problems/questions or approached step-by-step help. Being asked to explain how certain values are obtained, helped me understand how each step is related to another."

Drawbacks

As with any technique, there are some drawbacks in its implementation.

- Time-consuming.
- Need a good instructor/TA to student ratio.
- Paper heavy.

