

Mech 221: Computer Lab 2

Hand in the solutions to the three questions in the lab at the *end* of the lab.

In this lab, you will find numerical approximations to the differential equation

$$\frac{dy}{dt} = -y + 1$$

with initial condition $y(0) = 0$. The numerical method used will be the Forward Euler method that you hand wrote code for in pre-lab question #2. The exact solution to this problem is found in pre-lab question #1. Comparison of numerical and exact solutions will be done in lab questions #1 (comparison by plots) and #2 (error computation and convergence rate). You will modify your Forward Euler code to solve a different differential equation in question #3.

Question 1: Forward Euler Method

In this question, you will write an `.m` file that will numerically solve the differential equation given above. The `.m` file you will write is the one you prepared by hand as pre-lab question #2. You will run the code for different numbers of time steps N (different grid spacing h) and plot the different numerical solutions and the exact solution on the same graph. To do the plots, you will use the `hold on` command and `plot` command, as well as `plot` options as discussed in the pre-lab.

- Plot the exact solution to the differential equation above using a high number of points over the time interval $t = 0$ to $t = 1$. The command `linspace` from lab #1 will be useful here to make a vector of t values.
- Type in the `.m` file from your pre-lab question #2 that will do Forward Euler time stepping on this problem with $N = 4$ time steps (so 5 points for t and approximate y).
- Run your `.m` file
- On the same plot as the exact solution (use `hold on`), plot your numerical solution with $N = 4$ with a line and using your choice of marker style (circles, asterisks, etc.).

- Now take $N = 8, 16$, and numerically solve the differential equation using your code. Add plots of these solutions to the plot above. Make sure to use a different line style and/or different markers for your new plots.
- *Hand in the plot with the exact solution and the $N = 4, 8, 16$ approximate solutions for this question.*

Question 2: Analysis of Error in Forward Euler

In this question, you will compute the errors in the numerical solutions you found above, and estimate the convergence rate of the Forward Euler method.

- Compute the maximum error in each of your solutions $N = 4, 8, 16$ found in the previous question. Fill in the following table.

N	Max. Error
4	
8	
16	

- Identify the order of convergence of the Forward Euler Method. Read your online lecture notes section 2.3 if you need to be reminded of the definition of convergence order.
- *Hand in the completed table above and your estimate of the convergence order of the Forward Euler method.*

Question 3: Approximating a differential equation with no analytic solution

In this question, we will use the Forward Euler method to numerically solve a differential equation for $y(x)$ that we can not solve analytically. We will then use it to compute the value of the solution at $x = 1$ accurate to three significant figures.

The differential equation problem to approximate is

$$\frac{dy}{dx} = \sqrt{\frac{y}{y^2 + x^2}}, \quad y(0) = 1 \quad (1)$$

- Modify the .m file that you used in Question #1 so that your code will solve this problem on the interval $0 \leq x \leq 1$.
- Beginning with $N = 2$, double the value of N , until you have computed the value of $y(1)$ to 3 significant figures. Be careful to continue to increase N until you are really sure your answer is accurate enough.
- For $N = 16$ plot your approximate solution.
- *Hand in your .m file for this question, the plot above, the list of approximations for $y(1)$ for the N you tried, and the value of $y(1)$ you believe is accurate to 3 significant figures based on your computations.*