MECH 222 Computer Lab 7—Engine Optimization

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Learning Objectives

By the end of this lab, with reference to the Briefing Notes, you should be able to:

- derive and communicate thermodynamic fundamentals related to the Otto cycle,
- include more than one plot in the same Matlab figure (using `subplot`),
- share reference values of selected constants among a suite of related functions (using a combination of a setup file and global variables),
- create plots using a base-10 log scale on either or both axes,
- incorporate given Matlab functions with multiple outputs into high-level scripts (e.g., for plotting) after reading their source code,
- change the settings in `ode45` for trajectory plotting by following an explicit example, and
- automate the repeated execution of an existing script, where one of the script's parameters changes in each iteration (by wrapping a big `for`-loop around the script).

PRE-LAB ASSIGNMENT

The calculations you do for the pre-lab will be needed during the main lab activity. So keep a personal copy of your results for reference, as well as handing in a copy at the beginning of the session.

All four questions below refer to the states and transitions shown in Figure 1 of the Briefing Notes.

PL1: If the transition from state 1 to state 2 is reversible, and the piston mechanism contains an ideal gas, the path between these states must satisfy

$$\frac{dP}{P} = \frac{k-1}{PV}\,\delta Q - \frac{k}{V}\,dV. \qquad (*)$$

For the special case where $\delta Q = 0$ along the path, derive a relationship between P and V that holds along the path. (Yes, *derive*: don't just *quote*!)

PL2: Assume that in state 1, the gas pressure, volume, and temperature are

$$P_1 = 1.01 \times 10^5 \mathrm{N/m^2}, \qquad V_1 = 5.556 \times 10^{-4} \mathrm{m^3}, \qquad T_1 = 333\mathrm{K}.$$

Using $V_2 = 0.556 \times 10^{-4}$ m^3 with the assumptions in PL1, find the corresponding values in state 2. (Call these P_2, V_2, T_2.)

PL3: Approximate the transition from state 2 to state 3 as heat addition at constant volume. In this case, equation $(*)$ holds with $dV \equiv 0$ and integration gives

$$(P_3 - P_2) = \frac{k-1}{V}\left({}_2Q_3\right).$$

If the internal energy of the gas rises by 1800 J between states 2 and 3, find P_3, V_3, T_3.

PL4: Assume adiabatic and reversible expansion during the transition from state 3 to state 4. Find P_4, V_4, T_4.

ACTIVITY 0: *Log your session.*

Proceed as in Computer Lab 6 to put the scripts `startup.m`, `finish.m`, and `localLOGSAVENOW.m` into your working directory before you start Matlab. When Matlab starts, you should be presented with a short request to type your name and student number. Enter these and then just do your work as usual. If there is no such request, it means the `startup.m` file was not found. Try again to put it into the correct working directory. Ask the Lab TA if your attempts are unsuccessful.

Reserve a minute or two at the end of the lab to submit the archive file the session logging software will create on your computer. See Activity 6 below for details.

ACTIVITY 1: *Improve curves* $1 \to 2$ *and* $3 \to 4$ *in Figure 1.*

Have Matlab produce two accurate alternatives to Figure 1 of the Briefing Notes. In both cases, use the system description from Prelab PL2 and the energy input amount from Prelab PL3. Connect states 1–2 and 3–4 using accurate trajectories of the form derived in PL1; use a straight vertical line for the transition "$2 \to 3$". Use Matlab to label the axes and to put your name in the plot title. Both plots should present the same information, but with different axis scaling.

1.1 Show transitions $1 \to 2 \to 3 \to 4$ on axes where absolute pressure P in $\mathrm{N/m^2}$ is plotted against volume V in $\mathrm{m^3}$. (These are the axes in Figure 1.)

1.2 Show the same transitions on axes where the base-10 log of the pressure ratio P/P_1 is plotted against the base-10 log of the volume ratio V/V_1.

Suggestion. Write your plotting commands into a script instead of just typing them at the command prompt. This will give you a head start in producing related plots later.

Hand-in Checklist:

- ☐ Two plots better than Figure 1, with labelled axes and your name in their printed titles.
- ☐ A printout of the commands used to produce the figures.

ACTIVITY 2: *Assimilate given geometric information and heat-input strategy.*

2.1 Copy the function file `vdv.m` into your working directory. The function it defines takes a vector of crank angles (in radians) and returns the values of V and $dV/d\theta$ at these angles, where V is the gas volume in the cylinder. The prototype below shows how to use it:

```
[V,Vprime] = vdv(theta);
```

Read through the file to see how it works.

2.2 Copy the function file `qdq.m` into your working directory. The function it defines takes a vector of angles (in radians) and returns the values of Q and $\delta Q/d\theta$ at these angles, where Q is the heat transferred to the gas in the cylinder. The prototype below shows how to use it:

```
[Q,Qprime] = qdq(theta);
```

Read through the file to see how it works.

2.3 Acquire and run the script `cyclesetup`. Read through it to see how the following default values get initialized and turned into global variables:

$$\theta_s = -0.400, \quad \theta_b = 0.800, \quad \overline{Q} = 1800, \quad V_d = 5.00 \times 10^{-4},$$
$$r = 10.0, \quad R_1 = 3.00, \quad P_{\mathrm{amb}} = 1.01 \times 10^5, \quad k = 1.40.$$

Make sure you know the meaning and the units for each of these variables!

2.4 Draw a two-pane plot figure illustrating gas volume versus crank angle. (Recall Idea 4 in the Briefing Notes.) Use the default parameters shown in part 2.3. Put the volume V in the top pane, and the rate of change $V'(\theta) = dV/d\theta$ in the bottom. The horizontal axis should measure crank angle in radians, with values running from $-\pi$ to π. (Use this same interval for all θ-dependent plots in the lab.)

2.5 Draw a two-pane plot figure showing heat addition versus crank angle. The result should be a version of Figure 3 in the Briefing Notes, adapted for the parameters specific to your lab day. Plot $Q'(\theta)$ vs θ in the top pane, and show the cumulative heat added, $Q(\theta)$ vs θ, in the bottom. Use the default parameters shown in part 2.3 above, except for the spark angle θ_s and the burn duration angle θ_b. For these, use the numbers corresponding to your lab day in the setup below. As an extra challenge, the numbers are given in *degrees,* even though all the computations are done in *radians*. You will have to convert correctly to get reasonable plots.

θ_s (degrees) by lab day ...	Mon: -20,	Tue: 0,	Wed: 20,	Fri: -40;
θ_b (degrees) by lab day ...	Mon: 60,	Tue: 45,	Wed: 20,	Fri: 30.

Hand-in Checklist:

- ☐ Two 2-pane plots as described above, with labelled axes and your name in their printed titles.
- ☐ A printout of the commands used to produce the plots.

ACTIVITY 3: *Try a slower (more realistic) heat-input scenario in the simulated cylinder.*

Activity 2 gives us the gas's volume V and added heat Q as functions of the crank angle θ. The Briefing Notes explain why the corresponding values of pressure P must satisfy

$$P'(\theta) = \frac{k-1}{V(\theta)}\,Q'(\theta) - \frac{kP(\theta)}{V(\theta)}\,V'(\theta), \qquad P(-\pi) = P_{\text{amb}}.$$

We now use `ode45` to solve this initial-value problem for the function P. The basic requirement is a function that returns $dP/d\theta$ given (only) θ and $P(\theta)$. This function is given on Vista.

3.1 Copy the function file `dpdtheta.m` into your working directory. Read through it and compare with the right side of the differential equation above. Notice that the calculation involves functions V, V', and Q' we have already explored, and relies on all the physical and thermodynamic quantities stored in global variables.

3.2 Recall the parameter choices for your lab day specified in Activity 2. Use `ode45` to solve for $P = P(\theta)$ in the initial-value problem stated above. Request extra accuracy by stipulating a fine mesh of θ-values as nodes for the solution, and insisting on a small relative error tolerance, as follows:

```
thetanodes = linspace(-pi,pi,721);        % Lots of nodes
ODE45OPTIONS = odeset('reltol',1.E-6);    % Default reltol of 1.E-4 is sloppy
[ThetaList,PList] = ode45(@dpdtheta, thetanodes, Pamb, ODE45OPTIONS);
```

3.3 Use the given functions `vdv` and `qdq` to find $V(\theta)$ and $Q(\theta)$ at the θ-nodes of interest:

```
[VList,dVList] = vdv(ThetaList);
[QList,dQList] = qdq(ThetaList);
```

3.4 Make two plots in which the PV-trajectories computed in 3.3 are overlaid on the ideal PV-trajectories you produced in Activity 1. Use different colours and/or line styles to help distinguish the two traces. Get your name into the plot titles.

Hand-in Checklist:

- ☐ The two plots from Activity 1, overlaid with calculated trajectories as mentioned above.
- ☐ A printout of the commands used to produce the figures.

ACTIVITY 4: *Estimate thermodynamic efficiency.*

Write a script to do the following things. It will provide useful raw material for Activity 5.

4.1 Estimate the line integral W that gives the mechanical work done on the outside world for the trajectory calculated above. Recall that

$$W = \int P\,dV = \int_{\theta=-\pi}^{\pi} P(\theta)V'(\theta)\,d\theta.$$

The trapezoidal rule is a reasonable method; recall that Matlab has a built-in function named `trapz` that implements it.

4.2 Calculate the efficiency $\eta = W/\overline{Q}$ for the proposed scheme. Report three significant figures. Find the relative error (in percent) between the calculated efficiency and the ideal theoretical value, which is

$$\eta_{\max} = 1 - \frac{1}{r^{k-1}}.$$

Hand-in Checklist:

- ☐ A printout of your script.
- ☐ Your calculated value of the work W, the actual efficiency η, and the percentage discrepancy between η and $\eta_{\max}$.

ACTIVITY 5: *Optimize thermodynamic efficiency.*

Use the burn duration angle θ_b specified for your lab day in item 2.5.

5.1 Wrap the commands you used in Activities 3 and 4 in a `for`-loop that steps the spark angle θ_s through 91 equally-spaced choices from $-\theta_b$ to θ_b, inclusive. For each choice of θ_s, calculate the system trajectory and evaluate (and save) its thermodynamic efficiency. Plot the efficiency ratio $\eta/\eta_{\max}$ versus angle θ_s as a collection of dots. Identify the maximum efficiency value and the θ_s that achieves it.

NOTE: You can share the current value of θ_s with all elements of your environment by saving it in the global variable named `global_THETA_S`. This is the lab's only exception to the principle that all changes to global variables should be made in a single central location (the `cyclesetup` script).

5.2 Repeat Activity 3 using your day's value for θ_b and your optimal choice of θ_s. This should give two views of a PV-trajectory closer to the ideal one.

Hand-in Checklist:

- ☐ A plot of the efficiency ratio versus spark angle. Handwritten on the plot, (i) the optimum spark angle, (ii) the corresponding computed efficiency η.
- ☐ A printout of the `for`-loop used to do the work in 5.1.
- ☐ The two plots described in 5.2.

ACTIVITY 6: *Submit your Log File*

At the end of the lab, upload to Vista the log file produced by the scripts described in Activity 0. The name should have the general form `log12345678logNN-MAR-2011.mat`, with 12345678 replaced by your student number, and NN by the date. You have to do this from the computer you used to do your work: for many students, that means the machine in the PACE lab where you completed Activities 1–5.