Learning Science and Technology to Understand Your Domain and Improve Student Learning

Ken Koedinger
Human-Computer Interaction & Psychology
Carnegie Mellon University
Learning Science Success

**Cognitive Tutor Algebra course**

- Based on ACT-R theory & computational models of student problem solving
- Wide-spread use & evaluation
  - 500,000 students use daily at 2600 schools
  - 8 of 10 full-year field studies demonstrate significantly better student learning

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Intelligent tutoring goes to school in the big city.
How do deep structural representations form & facilitate transfer?

Relevant cognitive theory

• Analogical transfer via schema abstraction
  – Gick & Holyoak, 83; Gentner, Loewenstein et al, 09
• Implicit elements: Perceptual chunking, probabilistic grammars
  – Gobet, 02; Goldstone, Landy, & Son, 09
  – Lari & Young, 90; Li, Cohen, Koedinger, 10
• Explicit elements: Sense making, explanation-based learning, comparison of contrasting cases
• A cog architecture that can flexibly recompose abstracted knowledge components
  – ACT-R, ICARUS, Soar
Two Paths To Effective Instructional Improvement

• Performing domain-specific cognitive task analysis

• Applying domain-general principles of learning & instruction
Why is it so tempting to think we know a lot about effective teaching?
• You’ve had lots of experience with the English language.

• You might say you know English.

• But, do you know what you know?
Unpacking & repacking expertise: Chick sexing

- Experts don’t know, what they know
  - 98% accurate after years of on-the-job training

- Interviews led to design of “pictures in which critical features of various types were indicated”

- After just minutes of instruction, novices brought to 84% accuracy!

Cognitive Task Analysis

• Techniques to specify cognitive structures & processes associated with task performance
  – Think aloud, structured interviews
  – Newell & Simon: Knowledge-based computer simulations of human reasoning
Cognitive Task Analysis Improves Instruction

Studies: Traditional instruction vs. CTA-based

• Med school catheter insertion (Velmahos et al., 2004)
  – Sig greater pre to post gain; better with patients on all 4 measures (including needle insertion attempts!)

• Radar system troubleshooting (Schaafstal et al., 2000)
  – CTA group solved 2x malfunctions & in less time

• Spreadsheet use (Merrill, 2002)
  – Post-test: 89% vs. 64% in half of training time

• Lee (2004) meta-analysis: 1.7 effect size!
Isn’t knowledge analysis done for long-standing academic domains?

• Hasn't all this been worked out?

• Surely by now we understand the content of, say, Physics? or Algebra?
Difficulty Factors Assessment:
Discovering What is Hard for Students to Learn

Which problem type is most difficult for Algebra students?

Story Problem
As a waiter, Ted gets $6 per hour. One night he made $66 in tips and earned a total of $81.90. How many hours did Ted work?

Word Problem
Starting with some number, if I multiply it by 6 and then add 66, I get 81.90. What number did I start with?

Equation
\[ x \times 6 + 66 = 81.90 \]
Algebra Student Results:
Story Problems are Easier!

Percent Correct

70%
61%
42%
0%
20%
40%
60%
80%

Problem Representation

Story
Word
Equation


Typical textbook strategy

8. After buying donuts at Wholey Donuts, Laura multiplies the number of donuts she bought by their price of $0.37 per donut. Then she adds the $0.22 charge for the box they came in and gets $2.81. How many donuts did she buy?

\[0.37x + 0.22 = 2.81\]

\[0.37x = 2.59\]

\[\frac{0.37x}{0.37} = \frac{2.59}{0.37}\]

\[x = 7\]
Informal Strategies

5. Starting with some number, if I multiply it by .37 and then add .22, I get 2.81. What number did I start with?

2. After hearing that Mom won a lottery prize, Bill took the amount she won and subtracted the $64 that Mom kept for herself. Then he divided the remaining money among her 3 sons giving each $26.50. How much did Mom win?
Algebra Student Results: Story Problems are Easier!

- Story: 70%
- Word: 61%
- Equation: 42%


The foreign language of algebra: Difficulties with syntax & semantics

2. Solve for \( x \):

\[
x \times 25 + 10 = 110
\]

\[
\begin{array}{c}
10
\hline
-16
\end{array}
\]

\[
x \times 15 = 110
\]

\[
\begin{array}{c}
15
\hline
-15
\end{array}
\]

\[
x = 95
\]

2. Solve for \( x \):

\[
x \times 0.37 + 0.22 = 2.81
\]

\[
\begin{array}{c}
0.37
\hline
-0.22
\end{array}
\]

\[
0.15
\]

\[
0.22
\]

\[
0.59
\]

\[
2.81
\]
Expert Blind Spot:
Expertise can impair judgment of student difficulties

What’s behind expert blind spot?

• Blind spot results from limited memory of mostly implicit learning experiences

• Self-reflections on thinking are biased
  – More aware of verbally-mediated reasoning
    • More words => more thinking needed
  – Not aware of implicit processing & learning
    • Equations need to be “read” too
    • Fluent algebra language processing requires extensive implicit learning
But, symbols do help for more complex problems ...

More complex *multiple-unknown problems*

Story Problem
Roseanne just paid $38.24 for new jeans. She got them at a 15% discount. What was the original price?

Equation

\[ X - 0.15X = 38.24 \]

CTA used to design Algebra Cognitive Tutor & Text

Analyze real world problem scenarios

An experimental aircraft has sunk off the coast of South Africa at a depth of 12,790 feet. The military have located the aircraft and are in the process of raising it to the surface. It is currently 7625 feet below the surface and is being raised at the rate of 185 feet per hour. (Hint: Consider the direction above sea level to be positive)

1. How deep was the aircraft five hours ago?
2. How deep will the aircraft be five hours from now?
3. When did the military start raising the aircraft?
4. When will the aircraft reach the surface?

To write an expression, define a variable for the time from now and use this variable to write a rule for the depth of the aircraft.

Use graphs, graphics calculator

TIME Settings
-6 15 1
DEPTH Settings
-16,000 0 1,060

Use table, spreadsheet

<table>
<thead>
<tr>
<th>TIME</th>
<th>DEPTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>HOURS</td>
</tr>
<tr>
<td>Expression</td>
<td>H</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Use equations, symbolic calculator

-7625 + 185H = -12790
Add 7625
185H = -6,165
Divide by 185
H = -1,033/37

Tracked by knowledge tracing

Model tracing to provide context-sensitive Instruction

You have entered the given H in the wrong column of the worksheet
Cognitive Tutor Technology

Use cognitive model to individualize instruction

• Cognitive Model: A system that can solve problems in the various ways students can

\[ 3(2x - 5) = 9 \]

If goal is solve \( a(bx+c) = d \)
Then rewrite as \( abx + ac = d \)

\[ 6x - 15 = 9 \]

If goal is solve \( a(bx+c) = d \)
Then rewrite as \( abx + c = d \)

\[ 2x - 5 = 3 \]

If goal is solve \( a(bx+c) = d \)
Then rewrite as \( bx+c = d/a \)

\[ 6x - 5 = 9 \]

• Model Tracing: Follows student through their individual approach to a problem -> context-sensitive instruction
Cognitive Tutor Technology
Use cognitive model to individualize instruction

- Cognitive Model: A system that can solve problems in the various ways students can
  
  If goal is solve $a(bx+c) = d$
  Then rewrite as $abx + ac = d$

  Hint message: “Distribute $a$ across the parentheses.”
  Known? = 85% chance

  $3(2x - 5) = 9$
  $6x - 15 = 9$

  If goal is solve $a(bx+c) = d$
  Then rewrite as $abx + c = d$

  Bug message: “You need to multiply $c$ by $a$ also.”
  Known? = 45%

  $2x - 5 = 3$

- Model Tracing: Follows student through their individual approach to a problem -> context-sensitive instruction

- Knowledge Tracing: Assesses student's knowledge growth -> individualized activity selection and pacing
In vivo experiments that close the loop

• How does knowledge analysis lead to improved instructional design?
• Performance aids may not be learning aids
• Example *in vivo* experiments
  – *Inductive support*: Have students generalize formalisms from concrete solutions
  – *Symbolic language practice*

• *In vivo*: Principle-testing (change one thing) experiments within real courses
Drane & Route Plumbing Co. charges $42 per hour plus $35 for the service call.

Symbols & transfer

Common views
- Math & language cognition are quite different
- Analog problems support transfer
- Problem schemas are induced

Alternate view
- Elements of math learning engage language learning processing
- Parts of solutions can support transfer
- Modular hierarchical deep structure induced
Seeing Language Learning
Inside the Math:
Cognitive Analysis Yields Transfer

In Proceedings of the 32nd Annual Conference of the
<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>%Correct*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann is in a rowboat on a lake. She is 800 yards from the dock. She then rows for ( m ) minutes back towards the dock. Ann rows at a speed of 40 yards per minute. Write an expression for Ann's distance from the dock.</td>
<td>800-40m</td>
<td>40%</td>
</tr>
<tr>
<td><strong>Hint 1</strong>: Ann's distance from the dock is equal to the distance she started away from the dock minus the number of yards she rowed back toward the dock. <strong>Hint 2</strong>: The distance Ann has rowed is equal to the number of minutes multiplied by the number of yards per minute she rows.</td>
<td>800-40m</td>
<td>41%</td>
</tr>
<tr>
<td><strong>Problem to solve</strong> Ann is in a rowboat on a lake. She is 800 yards from the dock. She then rows for 3 minutes back towards the dock. Ann rows at a speed of 40 yards per minute. How far is Ann from the dock now?</td>
<td>680</td>
<td>63%</td>
</tr>
<tr>
<td>Ann #1: Ann is in a rowboat on a lake. She is 800 yards from the dock. She then rows ( y ) yards back towards the dock. Write an expression for Ann's distance from the dock.</td>
<td>800-( y )</td>
<td>62%</td>
</tr>
<tr>
<td>Ann #2: Ann is in a rowboat on a lake. Ann rows for ( m ) minutes back towards the dock. She rows at a speed of 40 yards per minute. Write an expression for the distance Ann has rowed.</td>
<td>40m</td>
<td>62%</td>
</tr>
</tbody>
</table>

* Heffernan & Koedinger, 1997
What’s hard about learning to symbolize?

Ann is in a rowboat on a lake. She is 800 yards from the dock. She then rows for m minutes back towards the dock. Ann rows at a speed of 40 yards per minute. Write an expression for Ann's distance from the dock.

Comprehension is not the key source of difficulty in translating story problems

Algebra is like a second language

In translating English to Greek, the hard part is not comprehending the English, but producing the Greek
Hypothesis: Substitution practice will aid algebra grammar learning & transfer

Substitute $40 \times m$ for $y$ in $800 - y$
Write the resulting expression.

Solution: $800 - 40m$

- Based on analogical transfer theory, such problems seem unlikely to help. Not similar to target problems.
- Alternatively, transfer may occur if
  - Probs support induction of recursive grammar patterns, like $\text{expr} \Rightarrow \text{expr op expr}$

2-step problem
Ms. Lindquist is a math teacher. Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches \textit{fewer boys} than girls. Write an expression for how many students Ms. Lindquist teaches.

1-step problem
Ms. Lindquist is a math teacher. Ms. Lindquist teaches 62 girls. Ms. Lindquist teaches \textit{b boys}. Write an expression for how many students Ms. Lindquist teaches.

Substitution problem
Substitute $62 - f$ for $b$ in $62 + b$
Write the resulting expression.
Substitution practice transfers to symbolization

<table>
<thead>
<tr>
<th>Condition</th>
<th>$n$</th>
<th>Pretest Mean (st error)</th>
<th>Instruct Mean (st error)</th>
<th>Filler Mean (st error)</th>
<th>Adj posttest Mean (st error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-step sym practice</td>
<td>154</td>
<td>.56 (.03)</td>
<td>.72 (.04)</td>
<td>.51 (.04)</td>
<td>.32 (.02)</td>
</tr>
<tr>
<td>Substitution practice</td>
<td>149</td>
<td>.57 (.03)</td>
<td>.52 (.04)</td>
<td>.72 (.04)</td>
<td>.39 (.02)</td>
</tr>
</tbody>
</table>

- Significant effect of condition ($p<.05$)
- Substitution transfers to story problems better than story problems themselves!
- Evidence for 1) algebra grammar
  2) decomposability of knowledge & instruction
Two Paths To Effective Instructional Improvement

• Performing domain-specific cognitive task analysis

• Applying domain-general principles of learning & instruction
What’s the best form of instruction? Two choices?

- More assistance vs. more challenge
  - Basics vs. understanding
  - Education wars in reading, math, science...
- Researchers like binary oppositions too. We just produce a lot more of them!
  - Massed vs. distributed (Pashler)
  - Study vs. test (Roediger)
  - Examples vs. problem solving (Sweller, Renkl)
  - Direct instruction vs. discovery learning (Klahr)
  - Re-explain vs. ask for explanation (Chi, Renkl)
  - Immediate vs. delayed (Anderson vs. Bjork)
  - Concrete vs. abstract (Pavio vs. Kaminski)
  - ...

Recommendations from DoE
Practice Guide

Organizing Instruction and Study to Improve Student Learning
A Practice Guide

Recommendation 1: Space learning over time.
To help students remember key facts, concepts, and knowledge, we recommend that teachers arrange for students to be exposed to key course concepts on at least two occasions—separated by a period of several weeks to several months. Research has shown that delayed re-exposure to course material often markedly increases the amount of information that students remember. The delayed re-exposure to the material can be promoted through homework assignments, in-class reviews, quizzes (see Recommendation 3), or other instructional exercises. In certain classes, important content is automatically

Recommendation 2: Interleave worked example solutions and problem-solving exercises.
When teaching mathematical or science problem solving, we recommend that teachers interleave worked example solutions and problem-solving exercises—literally alternating between worked examples demonstrating one possible solution path and problems that the student is asked to solve for himself or herself—because research has shown that this interleaving markedly enhances student learning.

Recommendation 7: Help students build explanations by asking and answering deep questions.
When students have acquired a basic set of knowledge about a particular topic of study and are ready to build a more complex understanding of a topic, we recommend that teachers find opportunities to ask questions and model answers to these questions. In order to help students build deep explanations of key concepts, by deep explanations we mean explanations that appeal to causal mechanisms, planning, well-reasoned arguments, and logic. Examples of deep explanations include those that inquire about causes and consequences of historical events, motivations of people involved in historical events, scientific evidence for particular theories, and logical justifications for the steps of a mathematical proof.

Examples of the types of questions that prompt deep explanations are why, why-not, how, what-if, how does X compare to Y, and what is the evidence for X? These questions and explanations can occur both during classroom instruction, class discussion, and during independent study.
How many options are there really? And what works best when?

What’s best?
- Focused practice
- Gradually widen
- Distributed practice

More help
- Basics
- Understanding

More challenge
- Concrete
- Mix
- Abstract

Study
- 50/50 Mix
- Test on problems

Study examples
- Concrete
- Abstract

Concrete
- Immediate
- Delayed
- No feedback

Mix
- Block topics in chapters
- Interleave topics
- Ask for explanations

Abstract
- Immediate
- Delayed
- No feedback

50/50 Mix
- Block topics in chapters
- Fade
- Interleave topics

Immediate
- Explain
- Mix

Delayed
- Explain
- Mix

No feedback
- Ask for explanations
Center-level effort needed to tackle this complexity

Cumulative theory development

Field-based basic research with microgenetic data collection

Derivation:
- 15 instructional dimensions
- 3 options per dimension
- 2 stages of learning

$\Rightarrow 3^{15*2}$ options

205,891,132,094,649
Many Pittsburgh Science of Learning Center studies in this space ...

- Researchers like binary oppositions too. We just produce a lot more of them!
  - Massed vs. **distributed** (Pashler)
  - Study vs. **test** (Roediger)
  - **Examples** vs. problem solving (Sweller, Renkl)
  - **Direct instruction** vs. discovery learning (Klahr)
  - Re-explain vs. **ask for explanation** (Chi, Renkl)
  - **Immediate** vs. **delayed** (Anderson vs. Bjork)
  - **Concrete** vs. **abstract** (Pavio vs. Kaminski)
  - ...

Example *In Vivo* Experiment on “Self-Explanation”

- **Self-explanation**: Have students explain steps in solutions

- *In vivo* experiments: Tightly controlled principle-testing experiment embedded in a real course

### Explanation Treatment Condition
(in computer tutor)

**Problem solving answers**

**Explanation by reference**

<table>
<thead>
<tr>
<th>Explanation</th>
<th>Treatment Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>External Angle &amp; Parallel Lines</strong></td>
<td></td>
</tr>
</tbody>
</table>

Given: OH || EC. If the measure of Angle SOR is a right angle, find the measure of Angle SRN.

<table>
<thead>
<tr>
<th>m∠SOR</th>
<th>m∠OSC</th>
<th>m∠OSR</th>
<th>m∠ESR</th>
<th>m∠SRN</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>90</td>
<td>45</td>
<td>135</td>
<td></td>
</tr>
</tbody>
</table>

**Reason**
- Given
- Int angles same side
- Angle bisection
- Angle addition

**Messages**

Some reasons dealing with parallel lines are highlighted in the Glossary. Which of these reasons is appropriate?

You can click on each reason in the Glossary to find out more.

**Glossary**

- Converse of Isosceles Triangle (Theorem)
- Isosceles Right Triangle
- Triangle Sum (Theorem)
- Linear Pair
- Linear Trio

**Parallel Lines**

- Corr. Angles Are Cong
- Alt. Int. Angles Are C.
- Alt. Ext. Angles Are C.
- Int. Angles on the Sa.

If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

**Example**: L₁ and L₂ are parallel lines, intersected by transversal T. ∠1 and ∠2 are alternate interior angles. If m∠1 is 37°, then m∠2 is also 37°.
Problem Solving Condition
(Control: Computer Tutor as it was)

Given: OH II EC. If the measure of Angle SOR is a right angle, find the measure of Angle SRN.

<table>
<thead>
<tr>
<th>m&lt;SOR</th>
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<tr>
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Parallel Lines --- Alt. Int. Angles Are C.
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Example: L₁ and L₂ are parallel lines, intersected by transversal T. ∠1 and ∠2 are alternate interior angles. If m∠1 is 37°, then m∠2 is also 37°.
Self-explanation improves understanding $\Rightarrow$ robust learning

Assistance Formula

- Formalizing instructional decision-making
  - General principles combined with CTA
- How to best apply an instructional principle
  - Spacing, examples, ...
- Need data to set parameters

\[ L = \frac{pS_b + (1 - p)F_b}{pS_c + (1 - p)F_c} \]

- \( L \) = long-term learning event efficiency
- \( p \) = probability of success
- \( S_b \) = long-term success benefit
- \( F_b \) = long-term failure benefit
- \( S_c \) = immediate success cost
- \( F_c \) = immediate failure cost
Rational Cognitive Task Analysis: Tools for Computational Modeling

• SimStudent: A computational modeling of student learning
  – Teach it by demonstrating correct actions & giving feedback on incorrect actions
  – Learns external symbol use by induction
  – Quick demo?

• Find at ctat.pact.cs.cmu.edu

Modeling reveals many complications of learning & transfer

- SimStudent errors reveal many challenges to inducing generalized knowledge
  - Can make errors in generalizing *where* to find info, *how* & *when* to perform actions

- After seeing example: $3x=9 \rightarrow x=3$
  - SimStudent successfully solves $4x=16$ with $x=4$
  - *How* error: $5x=15 \rightarrow x=5$
  - After feedback on this one problem, now successful on $7x=21$, $10x=35$, ...

- *When* error: $x/5=10 \rightarrow x=2$

- *Where* error: Gets stuck after $7x+4=25 \rightarrow 7x=21$
Summary

Two paths to improved instruction
1. Use CTA to uncover hidden keys to learning
   - Systematically collect student performance data to isolate KCs
2. Employ general instructional principles
   - Much has been discovered: worked examples, comparison, self-explanation ...
   - But much more to discover

• Beware of assistance dilemmas => Guide application of principles using CTA
Non-verbal *learning* processes and verbal *instruction*

- **Expert blind spot**
  - We remember verbal instruction
  - We don’t remember non-verbal learning processes that underlie much of expertise development

- **Non-verbal learning processes include**
  - Example-based induction/analogy, perceptual chunking & deep feature learning …

- **Verbal instruction influences non-verbal knowledge construction**
  - Worked examples & deep questions focus limited cognitive resources on deeper thinking rather than shallow doing
Conclusions

• Need more “learning by thinking”!
  – STEM class & homework is 95% problems, too much “learning by doing”
  – Should be 50% examples & deep questions

• We live with “learning” on a daily basis, but that doesn’t mean we understand it!
  – Need interdisciplinary science
    • to discover latent structure of domain knowledge
    • to better understand huge space of instructional options

Thank you!