Workshops and the first course in mathematical proof

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Course Description

MATH 220  Mathematical Proof

Sets and functions; induction; cardinality; properties of the real numbers; sequences, series, and limits. Logic, structure, style, and clarity of proofs emphasized throughout.

Course purpose:

- A gateway to pure mathematics
- An introduction to writing and understanding mathematical proofs – often for the first time in the students' academic lives.
Course Audience

- For most students the course is required by their major.
Sources of Difficulty

- Abrupt transition to mathematical abstraction
- Challenging material
- Variety of topics
- Expectations of mathematical rigour
- Large amount of mathematical formalism
- A lot of new notation
- Demands for clear and precise writing
- Need for problem-solving ability
Experiment – Spring 2012

- Replace every fourth lecture by a problem-solving workshop.
- Drop roughly ¼ of course content (analysis)

- Control section: Fall 2010, same instructor, selection of matching material. **No workshops.**
- Additional comparison: Fall 2011 – identical course material, textbook, course outline. Different instructor. **No workshops.**
Workshop Structure

- Problem-solving in groups of 2-4
- Homework due before workshop
- Workshop theme matches homework
- Quiz at the end of class, followed by group quiz
- Bi-weekly: every fourth lecture is replaced by a workshop
- Instructor and STLF give individual feedback, sometimes present one or two solutions on the board.
Let's Play

Pete's cat always sneezes before it rains. She sneezed today. "This means it will be raining, " thinks Pete.

Is he right?

"I think you should be more explicit here in step two."
Final Exam Marks: 2010(easier) vs 2012(harder)

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Marks adjusted to workshop topics only
Final Exam Marks: 2011 vs 2012

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Conclusion

- 7%-10% increase in A's
- Some decrease in F's (4%)
- Overall positive shift
- To reduce F's further: separate under-performing groups of students. Group quizzes easily reveal such groups.
- Enhance problem-solving and feedback components of workshops.

Let's do it again next year!
Student Response

- **Positive:** 29
- **Non-negative:** 7
  
  *Students request workshop improvement, such as more time or feedback*

- **Negative:** 7
  
  *Students are concerned about omitted material.*

*Source: end-of-term surveys, 77 total*
Sample Comments

- "As frustrating as they can be the group work seems to help"
- "The workshops were really helpful because it forces me to do questions and give immediate feedback on my progress. I also liked working with others as the interaction allowed me to learn from others and practice explaining things clearly to other people as well"
- "They were helpful as review periods and also helped me connect with other students to study with."
References

- **Logic Versus Pedagogy.** Morris Kline  
  *The American Mathematical Monthly*, 1970

- **Mathematical Circles: Russian Experience**  
  Authors: Dmitri Fomin, Sergey Genkin, and Ilia V. Itenberg  
  Published by: American Mathematical Society (1996)

- **Problem Solving: What I have learned from my students,**  
  James Sandefur, Math Department, Georgetown University